



## Introduction of computer algebra systems in electrical engineering education using Wolfram language on Raspberry pi

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**Abstract:** *This paper introduces the innovative use of symbolic calculation in field of computer science on low cost computers Raspberry pi with free software in education and engineering design. A new approach is presented for acquiring of basic knowledge in the field of solving electrical circuits using symbolic tools by applying computer algebra system. As an example of rapid learning, the analysis and solving of transient response in electrical circuits are illustrated. For this, it is needed a good knowledge of differential equations and set initial conditions. Solving can sometimes be exhausting when done by hand, but in complex circuits are almost impossible. Many numerical-based software tools can provide a graphical interpretation of the transitional response, but cannot provide an analytical result in close form solutions. The new methodology provides a way for students to learn faster solving of transient response of electrical circuits, and engineers gain a better insight into the processes that are running.*

**Keywords:** *symbolic computation, computer algebra system, transient response, software tools*

### 1. INTRODUCTION

Traditionally, for successful learning the field of computer and engineering sciences it is needed an excellent knowledge of mathematics. In particular, it is necessary to know solving of the integral, systems of linear equations, complex numbers and differential equations. Those learning it think it is disconnected, hard and uninteresting. Those trying to employ them think they do not know enough. Teachers are constantly striving to improve the methodology for the study of the mathematics' application in their fields. Yet using math is more important to the world than at any point in human history. So at one end we have got falling interest in education where math is applied, and at the other end we have got a more mathematical world, a more quantitative world than we ever have had. Today, there are numerous mathematical software tools such as Matlab and Mathematica, which students and engineers in computer science can be used to solve complex mathematical problems [1], [2]. Instead of exhausting the students to acquire routine in solving mathematical problems, they are taught the basic theoretical concepts, and explain to them

how to use this knowledge in setting problems that can meet in practice. Although the traditional orientation, such as electronics, telecommunications, energy and automation, are based on hardware solutions and components, the needs of companies, which employ engineers, are primarily in the fields of programmable hardware and the computer's application. Therefore, it tends to the basic courses are adapted for successfully mastering the skills necessary for engineer's competences. From the above facts, this paper introduces a methodology that the students apply software based on mathematics in faster and deeper understanding, and solving the electric circuits, without high level of mathematical knowledge. Relying on the classical approach in learning and applying mathematical tools in engineering sciences, distinguished four stages in learning methodology [3], [4]. The first one represents the formulation of the problem. The second involves the transfer of settings from a position of real-world problem into a mathematical problem. Third stage is followed by calculating. Finally, obtained answer in a mathematical form should be returned to the real world. Third step spends the most of time teaching students to do it by hand. However, it is a step that computers can work better than any man after many years of practice. Instead, it is necessary to use a computer for this step, and force students to be more assiduous in learning of the first, second and fourth steps - problem conceptualizing, its application, and force teachers to teach them how. In this sense, a variant of low cost computers, named Raspberry Pi [5], are appeared on the market, recently. Its performance satisfies the characteristics of a medium PC and allows access to each person to use. Free software is installed, and to the software package Mathematica which is one of the best programming language for symbolic computation, named Wolfram language [1]. All together, the Raspberry pi, operating systems on the Raspberry pi and Wolfram language are implemented in one entirety, making the integrated computer algebra system. So far, the computer algebra system included only software support on whose platform the mathematical tools are used. Now, it becomes an integral part of the hardware, especially with the aforementioned low-cost computers with free software.

## **2. MODELLING OF ELECTRICAL CIRCUITS AND SYSTEMS**

To analyze the properties of the elements or system, mathematical models are used often. Modelling is the process of presenting a physical elements or system in a manner that enables the use of mathematical expressions [6]. Simplification of the model is carried out by adopting a number of assumptions that do not affect the essential properties of the element, a single analysis gives good results that show the essence and the most important features [7]. The electrical circuit represents the connection of two or more elements. Connecting element is made conductive. The most often, the resistances of conductives are ignored. Node of circuit shows that the two connections are short-circuited and represents the connecting point of two branches and two or more circuit elements. The loop is any closed path through the circuit so that through one node can go only once. The term of contour is used for loop that does not contain any other loop.

### **2.1. Circuits with variable currents**

Changes connections between the elements in electrical circuits are modelled by opening and closing a switch very often. Theoretically, analysis of circuits means that the phenomenon known from infinity to the observed moment, for example, if the element is modelled by integral. To avoid too complicated mathematical apparatus, the appearances in electrical circuits are modelled from a certain moment. It represents switching on an

element that connects with branch of circuit, and from that moment to observe what happens to the voltages and currents in the circuit. Practical significance is the phenomena that occurs from the moment of change, but until all the voltages and currents cease to change, i.e. reach a value that is practically unchangeable. This paper introduces a new approach in solving of electrical circuits with transient response using mathematical models, which are based on current changes. In practice, these changes gradually changed from one value to another [8], [9].

Such an analysis makes easier calculation and the essence of the changes that are occurred still reasonably accurate. In the analysis of transient response, two passive elements, capacitors and inductors, have a dependency on the voltage and current that are proportional to derivation and integral, while the third element, the resistor, has a constant relationship between voltage and current. All three elements are linear, because the relation between voltage and current is represented by the linear differential equations.

## 2.2. Differential equations of second order

For successful solving of voltage and current circuits with one resistor, one capacitor and one coil, in situations when voltage or current independent sources are changed, it is necessary to know mathematical equations solving of differential equation of second order.

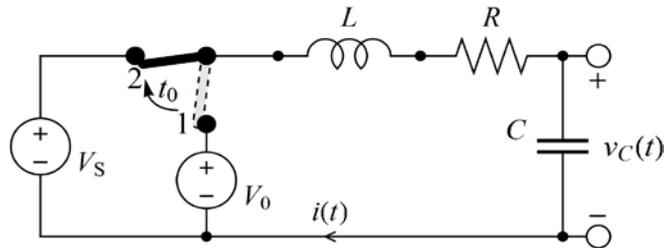
It is understood that the second order differential equation has constant coefficients:

$$\frac{d^2x(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_2x(t) = f(t) \quad (1)$$

Generally, it is needed to find solutions of previous equation  $x(t)$ . The variable can be represented by the sum of two solutions, one of which is called a forced solution, and another natural solution. The coefficients  $a_1$  and  $a_2$  are constants and the function  $f(t)$  can be any function of time. For the current and voltage in a transitional response, the function  $f(t)$  is constant and can be equal to zero. For both solutions, it is necessary to implement a number of steps and eventually write the final solution based on the analysis of specific cases.

## 3. SERIAL CONNECTION OF COIL, RESISTOR AND CAPACITOR

When capacitor and inductor are present in the same time in the circuit, then circuit that is more complex is obtained. These are circuits composed by sources, resistors, capacitors and inductors. Consider now the circuit in Figure 1. This circuit represents a serial connection of voltage sources, coil, resistor and capacitor. Now of  $t_0$  there is a change switch position, so that the value of the DC voltage source changes. Up to the moment of  $t_0$  sufficient time has elapsed to steady state, and there are no changes of voltages or currents in the circuit. Since the capacitor is serial linked with other elements, the current through all of the elements is zero. Since the voltage on the resistor and the coil is zero, the voltage on the capacitor will be equal to the voltage by the independent source. Accordingly, the initial value of the coil current is zero, and the initial value of the voltage on the capacitor is  $V_0$ . At the moment of  $t_0$  the switch is switched from position 1 to position 2, and then the resistor and the coil between two nodes are on a different potential, on the one side this is voltage of the new voltage source, and the other side this is voltage on the capacitor, which can not to change the value immediately. In addition, the current through the coil, which is the current through all the elements, cannot be changed immediately.



**Figure 1.** Electrical circuit of second order with serial link of coil, resistor and capacitor.

In this case, solving of the transient response is done by writing the general form of differential equations in Wolfram language. The symbol  $x$  represents the current through all elements of circuits by applying equation (1). Square brackets  $[ ]$  are used to highlight the independent variable, it is the time  $t$  in this case. Designation  $f$  is any function, but in this analysis, it is constant or, more often equals zero. The first and second copy ( $'$  and  $''$ ) to denote apostrophes immediately after the symbol for the time function  $x$ . Blank sign between the expressions is used instead of a sign for multiplication, double equality ( $==$ ) is used to emphasize that the left side is equal to the right, and one equal sign ( $=$ ) is used to assign some expression to a symbol - in this case it is the `eq1`, which assigns the entire expression that describes the differential equation:

$$\text{eq1} = x''[t] + a1 x'[t] + a2 x[t] == f$$

Constants ( $a1$  and  $a2$ ) are assigned in the equation in accordance with the established equations:

$$a1 = r/L;$$

$$a2 = 1/(L c);$$

Resistance, capacitance and inductance are marked with symbols  $r$ ,  $L$  and  $c$ . The initial condition  $x[0]$  is assigned to the symbol of `cond1`.

$$\text{cond1} = x[0] == 0;$$

The command that finds the solution of differential equations is `DSolve`. Arguments of this command are the symbols that contain a description of the differential function `eq1`, if the initial conditions are known `cond1`, symbolic code for a function that is obtained by solving the differential equation  $x[t]$  and the symbol for the independent variable  $t$ :

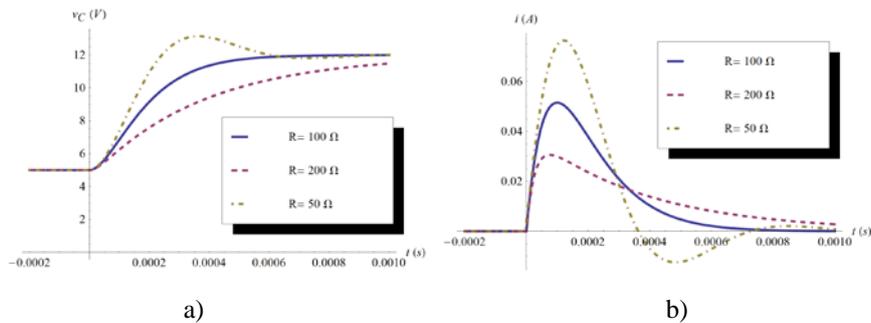
$$\text{DSolve}\{\{\text{eq1}, \text{cond1}\}, x[t], t\}$$

In this example, the solution will contain one specific constant defined as a symbol of `C[1]`. In order to determine it is needed to do integration of current according to the formula for the voltage of capacitor, to determine limit when the independent variable tends to infinity. By equating the limit with the value of voltage of capacitor with end of all changes, unknown constant `C[1]` is easily determined. For all three cases, the software determines the symbolic solution, where values of components are  $L=5$  mH,  $C=2$   $\mu$ F,  $R=100$   $\Omega$  for critically dimmed solution ( $R=200$   $\Omega$  to dim the solution,  $R=50$   $\Omega$  for undimmed solution)  $V_S=12$  V,  $V_0=5$  V. The response of the close form in Wolfram

language is shown as follows:

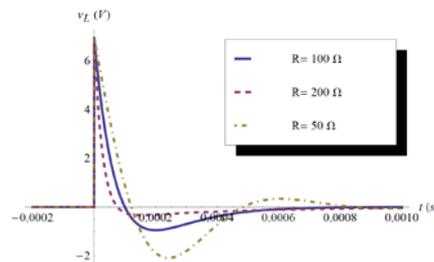
$$\{1400 e^{(-10000 \tau)} \tau\}$$

In Figure 2a) shows a change in voltage on the capacitor, and Figure 2b) shows current that flows through all elements of the circuit. Current has been zero to the moment of change of the switch, and current asymptotically approaches zero. The current is approximately equal to zero already after 1 ms. It seems that undimmed solution has an oscillatory character, and that is likely to approach zero when the resistance value is equal to dim critical condition.



**Figure 2.** Voltage on capacitor a) and current through elements of circuit b) for  $C=2 \mu F$ ,  $L=2mH$ ,  $V_S=12V$ ,  $V_0=5V$ .

The voltage on the coil can be determined when the voltage of the voltage source subtracted voltages across the resistor and capacitor. This example demonstrates that it is enough to know the basic laws of electrical engineering (Ohm's law, Kirchhoff's first law, Kirchhoff's second law, the expressions that define the relationship between the voltage and current of passive elements), the limit value when there is no changes of voltages and currents in the circuit (the coil is replaced by a short connection, the capacitor is replaced by an open connection).



**Figure 3.** Voltage on coil and capacitor a) and current through elements of circuit b) for  $C=2 \mu F$ ,  $L=2mH$ ,  $V_S=12V$ ,  $V_0=5V$ .

Differential equation should be entered in the manner as was done in this case, and the software will find a solution in close form as if the execution carried out by hand (software will determine which of the three cases is really happening). All figures are drawn based on the derived expressions of software in this paper.

#### 4. CONCLUSION

Application of new technologies in education aims to speed up the learning process. This is especially important in engineering sciences because of the great needs of the economy in information technology to quickly obtain expert. Concerning this, teaching materials, which are studied and where mathematics is present, have aim a deeper and faster understanding of engineering fields by students. Since much of the time spent in learning to perform derivation of formula, it can have a consequence that the student makes a mistake or do not understand the problem solved. This paper introduces the use of symbolic calculation in the field of computer science at cheap Raspberry Pi computers with free software. A new way for the acquisition of basic knowledge in the field of solving electrical circuits using symbolic tools using computer algebra system is presented. As an example of rapid learning presents the analysis and resolution of transient response in electrical circuits. Illustration of writing equations in the software package using Wolfram language to write code skips the part of the calculation that takes the most of time. Thus allowing the student to quickly get ready-made solutions.

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